OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4762**

Mechanics 2

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

*   Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully and make sure that you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a graphical calculator in this paper.
*   Final answers should be given to a degree of accuracy appropriate to the context.
*   The acceleration due to gravity is denoted by $g\,\text{m\,s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets **[ ]** at the end of each question or part question.
*   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
*   The total number of marks for this paper is **72**.
*   This document consists of **8** pages. Any blank pages are indicated.

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Turn over

1   Two sledges P and Q, with their loads, have masses of 200 kg and 250 kg respectively and are sliding on a horizontal surface against negligible resistance. There is an inextensible light rope connecting the sledges that is horizontal and parallel to the direction of motion.

Fig. 1 shows the initial situation with both sledges travelling with a velocity of $5\mathbf{i}\,\mathrm{m\,s}^{-1}$.
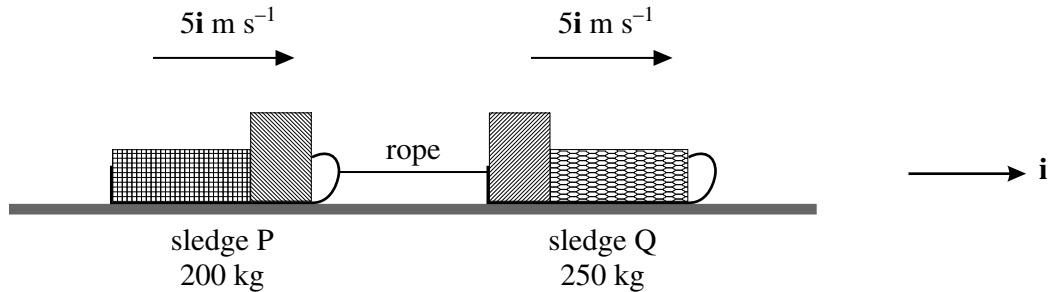

Fig. 1

A mechanism on Q jerks the rope so that there is an impulse of $250\mathbf{i}\,\mathrm{N\,s}$ on P.

  (i) Show that the new velocity of P is $6.25\mathbf{i}\,\mathrm{m\,s}^{-1}$ and find the new velocity of Q.   **[4]**

There is now a direct collision between the sledges and after the impact P has velocity $4.5\mathbf{i}\,\mathrm{m\,s}^{-1}$.

 (ii) Show that the velocity of Q becomes $5.4\mathbf{i}\,\mathrm{m\,s}^{-1}$.

      Calculate the coefficient of restitution in the collision.   **[6]**

Before the rope becomes taut again, the velocity of P is increased so that it catches up with Q. This is done by throwing part of the load from sledge P in the $-\mathbf{i}$ direction so that P's velocity increases to $5.5\mathbf{i}\,\mathrm{m\,s}^{-1}$. The part of the load thrown out is an object of mass 20 kg.

(iii) Calculate the speed of separation of the object from P.   **[5]**

When the sledges directly collide again they are held together and move as a single object.

(iv) Calculate the common velocity of the pair of sledges, giving your answer correct to 3 significant figures.   **[2]**
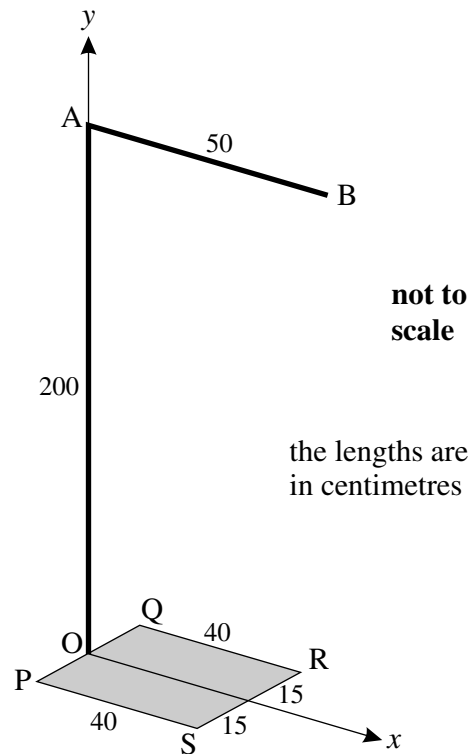
2

Fig. 2

Fig. 2 shows a stand on a horizontal floor and horizontal and vertical coordinate axes O*x* and O*y*. The stand is modelled as

- a thin uniform rectangular base PQRS, 30 cm by 40 cm with mass 15 kg; the sides QR and PS are parallel to O*x*,
- a thin uniform vertical rod of length 200 cm and mass 3 kg that is fixed to the base at O, the mid-point of PQ and the origin of coordinates,
- a thin uniform top rod AB of length 50 cm and mass 2 kg; AB is parallel to O*x*.

Coordinates are referred to the axes shown in the figure.

  (i) Calculate the coordinates of the centre of mass of the stand. **[5]**

A small object of mass 5 kg is fixed to the rod AB at a distance of 40 cm from A.

 (ii) Show that the coordinates of the centre of mass of the stand with the object are (22, 68). **[2]**

The stand is tilted about the edge PQ until it is on the point of toppling. The angle through which the stand is tilted is called 'the angle of tilt'. This procedure is repeated about the edges QR and RS.

(iii) Making your method clear, determine which edge requires the smallest angle of tilt for the stand to topple. **[4]**

The small object is removed. A light string is attached to the stand at A and pulled at an angle of 50° to the downward vertical in the plane O*xy* in an attempt to tip the stand about the edge RS.

(iv) Assuming that the stand does not slide, find the tension in the string when the stand is about to turn about the edge RS. **[7]**

3 Fig. 3 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, CD, DA and BD. The rods are freely pin-jointed to each other at A, B, C and D and to a vertical wall at A. ABCD is a parallelogram with AD horizontal and BD vertical; the dimensions of the framework, in metres, are shown. There is a vertical load of 300 N acting at C and a vertical wire attached to D, with tension T N, holds the framework in equilibrium. The horizontal and vertical forces, X N and Y N, acting on the framework at A due to the wall are also shown.
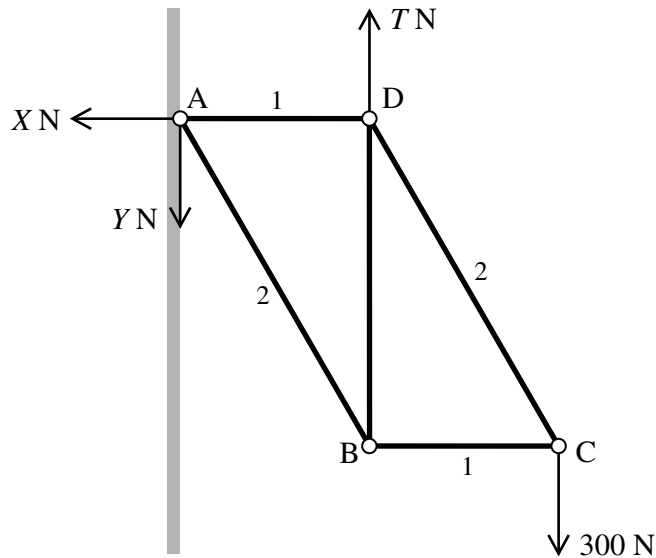

Fig. 3

   (i) Show that $T = 600$ and calculate the values of X and Y.   **[4]**

  (ii) Draw a diagram showing all the forces acting on the framework, and also the internal forces in the rods.   **[2]**

 (iii) Calculate the internal forces in the five rods, indicating whether each rod is in tension or compression (thrust). (You may leave answers in surd form. Your working in this part should correspond to your diagram in part **(ii)**.)   **[9]**

Suppose that the vertical wire is attached at B instead of D and that the framework is still in equilibrium.

 (iv) Without doing any further calculations, state which four of the rods have the same internal forces as in part **(iii)** and say briefly why this is the case. Determine the new force in the fifth rod. **[4]**

4 A box of mass 16 kg is on a uniformly rough horizontal floor with an applied force of fixed direction but varying magnitude P N acting as shown in Fig. 4. You may assume that the box does not tip for any value of P. The coefficient of friction between the box and the floor is μ.
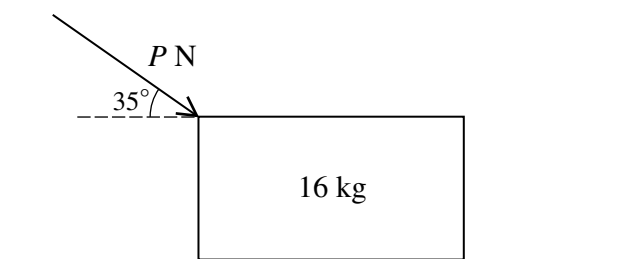

Fig. 4

Initially the box is at rest and on the point of slipping with $P = 58$.

   (i) Show that μ is about 0.25. **[5]**

In the rest of this question take μ to be exactly 0.25.

The applied force on the box is suddenly increased so that $P = 70$ and the box moves against friction with the floor and another horizontal retarding force, S. The box reaches a speed of $1.5\,\text{m s}^{-1}$ from rest after 5 seconds; during this time the box slides 3 m.

  (ii) Calculate the work done by the applied force of 70 N and also the average power developed by this force over the 5 seconds. **[4]**

 (iii) By considering the values of time, distance and speed, show that an object starting from rest that travels 3 m while reaching a speed of $1.5\,\text{m s}^{-1}$ in 5 seconds cannot be moving with constant acceleration. **[2]**

The reason that the acceleration is not constant is that the retarding force S is not constant.

  (iv) Calculate the total work done by the retarding force S. **[7]**

GCE

Mark Scheme for June 2010

Q 1		mark		sub
(i)	For P $200 \times 5 + 250 = 200 v_P$ $v_P = 6.25$ so $6.25\mathbf{i}$ m s^{-1} For Q $250 \times 5 - 250 = 250 v_Q$ $v_Q = 4$ so $4\mathbf{i}$ m s^{-1}	M1 E1 M1 A1	Award for I-M Accept no $\mathbf{i}$ and no units Must have impulse in opposite sense Must indicate direction. Accept no units.	 4
(ii)	$\mathbf{i}$ direction positive PCLM: $2250 = 200 \times 4.5 + 250 w_Q$ $w_Q = 5.4$ so $5.4\mathbf{i}$ m s^{-1} NEL: $\frac{w_Q - 4.5}{4 - 6.25} = -e$ $e = 0.4$	 M1 F1 E1 M1 A1 A1	 PCLM used. Allow error in LHS FT from (i) Any form. FT only from (i) NEL . Allow sign errors Signs correct. FT only from (i) cao	 6
(iii)	$\mathbf{i}$ direction positive Suppose absolute vel of object is $-V\mathbf{i}$ $200 \times 4.5 = -20V + 180 \times 5.5$ $V = 4.5$ speed of separation is $5.5 + 4.5 = 10$ m s^{-1}	 M1 B1 A1 A1 F1	 Applying PCLM. All terms present. Allow sign errors. Correct masses All correct (including signs) FT **their** V.	 5
(iv)	$180 \times 5.5 + 250 \times 5.4 = 430W$ $W = 5.4418\ldots$ so $5.44\ \mathbf{i}$ m s^{-1} (3 s. f.)	M1 A1	Using correct masses and velocities cao	 2
				17

Q 2		mark		sub
(i)	$20\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=15\begin{pmatrix}20\\0\end{pmatrix}+3\begin{pmatrix}0\\100\end{pmatrix}+2\begin{pmatrix}25\\200\end{pmatrix}$	M1	Method to obtain at least 1 coordinate	
		B1	'100' or '25' correct	
		A1	Either one RHS term correct or one component of two RHS terms correct	
	$\bar{x}=17.5$	A1		
	$\bar{y}=35$	A1		5
(ii)	$25\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}350\\700\end{pmatrix}+5\begin{pmatrix}40\\200\end{pmatrix}$	M1	Using (i) or starting again	
	so $\bar{x}=22$, $\bar{y}=68$	E1	Clearly shown.	2
(iii)	We need the edge that the $\bar{x}$ position is nearest	M1	This may be implied	
	$\bar{x}=22$; distances are 22 to PQ, 18 to SR	B1	One distance correct	
	15 to QR	B1	All distances correct	
	so edge QR	A1		4
(iv)	Moments about RS	M1	Moments about RS attempted	
		B1	Use of weight not mass below. FT mass from here	
	In sense xOy			
	$T\sin 50\times 200 - T\cos 50\times 40$	M1	Attempt to find moment of T about RS, including attempt at resolution. May try to find perp dist from G to line of action of the force.	
		A1		
	$-20g\times(40-17.5)=0$	B1	$40-17.5$	
		A1	All correct allowing sign errors	
	$T=34.5889\ldots$ so 34.6 N (3 s. f.)	A1	cao (except for use of mass)	7
				18

Q 3		mark		sub
(i)	a.c. moments about A $1 \times T - 2 \times 300 = 0$ so $T = 600$ Resolving $\rightarrow X = 0$ $\uparrow T - Y = 300$ so $Y = 300$	E1 B1 M1 A1	Justified	4
(ii)	Diagram *The working below sets all internal forces as tensions; candidates need not do this.*	B1 B1	All external forces marked consistent with (i) All internal forces with arrows and labels	2
(iii)	Let angle DAB be θ. $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ A $\uparrow$ $-300 - T_{AB}\sin\theta = 0$ so $T_{AB} = -200\sqrt{3}$ so force is $200\sqrt{3}$ (C) A $\rightarrow$ $T_{AD} + T_{AB}\cos\theta = 0$ so $T_{AD} = 100\sqrt{3}$ so force is $100\sqrt{3}$ (T) C $\uparrow$ $T_{CD}\sin\theta - 300 = 0$ so $T_{CD} = 200\sqrt{3}$ so force is $200\sqrt{3}$ (T) C $\leftarrow$ $T_{BC} + T_{CD}\cos\theta = 0$ so $T_{BC} = -100\sqrt{3}$ so force is $100\sqrt{3}$ (C) B $\uparrow$ $T_{AB}\sin\theta + T_{BD} = 0$ so $T_{BD} = 300$ so force is 300 (T)	B1 M1 M1 A1 F1 F1 F1 F1 F1	Or equivalent seen Attempt at equilibrium at pin-joints 1 equilib correct, allowing sign errors All T/C consistent with their calculations and diagrams	9
(iv)	AD, AB, BC, CD 300 N, *X* and *Y* not changed. Equilibrium equations at A and C are not altered B $\uparrow$ $T_{AB}\sin\theta + T'_{BD} + 600 = 0$ so $T'_{BD} = -300$ so force is 300 (C)	B1 E1 M1 A1	 C not needed. [If 300 N (C) given WWW, award SC1 (NB it must be made clear that this is a compression)]	4
				19

Q 4		mark		sub
(i)	Let friction be F N and normal reaction R N $F_{\max} = 58\cos 35$ $R = 16g + 58\sin 35$ $F_{\max} = \mu R$ so $\mu = 0.249968\ldots$ about 0.25	B1 M1 A1 M1 E1	Need not be explicit Both terms required.	5
(ii)	WD is $70\cos 35 \times 3 = 210\cos 35$ so $172.0219\ldots = 172$ J (3 s. f.) Average power is WD/time so $34.4043\ldots = 34.4$ W (3 s. f.)	M1 A1 M1 A1	Use of WD $= Fd$. Accept $\cos 35$ omitted. cao	4
(iii)	Using the constant acceleration result $s = \frac{1}{2}(u+v)t$ with $s = 3$, $u = 0$, $v = 1.5$ and $t = 5$ we see that $3 \neq \frac{1}{2}(0+1.5)\times 5 = 3.75$	M1 E1	Attempt to substitute in *suvat* (sequence) Conclusion clear	2
(iv)	$172.0219\ldots$ $= \frac{1}{2}\times 16 \times 1.5^2$ $+0.25\times(16g + 70\sin 35)\times 3$ $+ $ WD so WD by S is $6.30916\ldots$ so 6.31 J (3 s. f.)	M1 M1 A1 M1 A1 A1 A1	Using W-E equn, allow 1 missing term KE term attempted correct Attempt at using new F in $F_{\max} = \mu R$ All correct cao	7
				18